

The splitting of the one-body potential in spin-polarized isospin-symmetric nuclear matter

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Spin-polarized symmetric nuclear matter is studied within the Dirac-Brueckner-Hartree-Fock approach. We pay particular attention to the difference between the one-body potentials of upward and downward polarized nucleons. This is formally analogous to the “Lane potential” for isospin-asymmetric nuclear matter. We point out the necessity for additional information on this fundamentally important quantity and suggest ways to constrain it.

I. INTRODUCTION

Describing the properties of spin and isospin *symmetric* nuclear matter still presents considerable intellectual challenges. For instance, the physical pictures of the underlying one-particle fields are very different in relativistic and non-relativistic approaches. In relativistic models, saturation mechanisms are introduced through negative energy Dirac states, whereas non-relativistic approaches must be implemented with three-body forces (TBF) in order to describe saturation properties correctly. Although the relation between the two philosophies seems to be understood in terms of TBF of the “Z-diagram” type, saturation details can be quite different in the two frameworks.

When other aspects are considered, such as spin and/or isospin polarized states of nuclear matter, conclusions become even more model dependent and available constraints are very limited. The magnetic properties of neutron/nuclear matter have been studied extensively with a variety of theoretical methods [1-28]. In a previous calculation [29], we have investigated spin-polarized pure neutron matter (NM). Such system has gathered much attention lately, in relation to the issue of possible ferromagnetic instabilities. Also, the possibility of strong magnetic fields in the interior of neutron stars makes the study of polarized NM important and timely.

Although these are very exciting issues, there are other motivations for studies of polarized matter. Here, for instance, we will focus on the spin degrees of freedom of symmetric nuclear matter (SNM), having in mind a terrestrial scenario as a possible “laboratory”. We will pay particular attention to the spin-dependent *symmetry potential*, namely the gradient between the single-nucleon potentials for upward and downward polarized nucleons. The interest around this quantity arises because of its natural interpretation as a spin dependent nuclear optical potential, defined in perfect formal analogy with the Lane potential [30] for the isospin degree of freedom in isospin-asymmetric nuclear matter (IANM).

Concerning optical potential analyses, to the best of our knowledge, spin degrees of freedom have not been given much attention, possibly due to the increased difficulties in obtaining empirical constraints as compared to the unpolarized system. Another way to access information related to the spin dependence of the nuclear interaction in nuclear matter is the study of collective modes such as spin giant resonances (SGR). However, those are not easily observed with sufficient strength [28, 31].

In summary, if constraints on the isospin-dependent properties of the nuclear equation of state (EoS) are still scarce, those on the spin dependence are even more so, and should be pursued along with theoretical predictions of the spin-dependent nucleon field. What makes this issue particularly interesting is that spin degrees of freedom and relativity are inherently tied with each other. Thus, comparison between relativistic and non-relativistic predictions should be insightful.

This paper is organized as follows: In the next Section, we review the main aspects of the formalism. We will then demonstrate the splitting of the one-body potential in spin-asymmetric matter and discuss its significance. Our conclusions are summarized in the last Section.

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II. FORMALISM

Our calculation is microscopic and treats the nucleons relativistically. Within the Dirac-Brueckner-Hartree-Fock (DBHF) method, the interactions of the nucleons with the nuclear medium are expressed as self-energy corrections to the nucleon propagator. That is, the nucleons are regarded as “dressed” quasi-particles. Relativistic effects lead to an intrinsically density-dependent interaction which is consistent with the contribution from TBF typically employed in non-relativistic approaches. The advantage of the DBHF approximation is the absence of phenomenological TBF to be extrapolated at higher densities from their values determined through observables at normal density.

The starting point of any microscopic calculation of nuclear structure or reactions is a realistic free-space nucleon-nucleon interaction. A realistic and quantitative model for the nuclear force with reasonable foundations in theory is the one-boson-exchange (OBE) model [32]. Our standard framework consists of the Bonn B potential together with the DBHF approach to nuclear matter. A detailed description of our application of the DBHF method to nuclear, neutron, and asymmetric matter can be found in a recent review of our work [33].

Similarly to what we have done to describe isospin asymmetries of nuclear matter, the single-particle potential is the solution of a set of coupled equations,

$$U_u = U_{ud} + U_{uu} \quad (1)$$

$$U_d = U_{du} + U_{dd} , \quad (2)$$

to be solved self-consistently along with the two-nucleon G -matrix. In the above equations, u and d refer to spin-up and spin-down polarizations, respectively, and each $U_{\sigma\sigma'}$ term contains the appropriate (spin-dependent) part of the interaction, $G_{\sigma\sigma'}$. More specifically,

$$U_{\sigma}(\vec{p}) = \sum_{\sigma'=u,d} \sum_{q \leq k_F^{\sigma'}} < \sigma, \sigma' | G(\vec{p}, \vec{q}) | \sigma, \sigma' > , \quad (3)$$

where the second summation indicates integration over the Fermi sea of spin-up (or spin-down) nucleons, and

$$\begin{aligned} < \sigma, \sigma' | G(\vec{p}, \vec{q}) | \sigma, \sigma' > = \sum_{L, L', S, J, M, M_L} < \frac{1}{2}\sigma; \frac{1}{2}\sigma' | S(\sigma + \sigma') > < \frac{1}{2}\sigma; \frac{1}{2}\sigma' | S(\sigma + \sigma') > \\ & \times < L M_L; S(\sigma + \sigma') | J M > < L' M_L; S(\sigma + \sigma') | J M > \\ & \times i^{L'-L} Y_{L', M_L}^*(\hat{k}_{rel}) Y_{L, M_L}(\hat{k}_{rel}) < L S J | G(k_{rel}, K_{c.m.}) | L' S J > . \end{aligned} \quad (4)$$

The notation $< j_1 m_1; j_2 m_2 | j_3 m_3 >$ is used for the Clebsh-Gordan coefficients. Clearly, the need to separate the interaction by spin components brings along angular dependence, with the result that the single-particle potential depends also on the direction of the momentum. The G -matrix equation is solved using partial wave decomposition and the matrix elements are then summed as in Eq. (4) to provide the new matrix elements in the uncoupled-spin representation needed for Eq. (3). Furthermore, the scattering equation is solved using relative and center-of-mass coordinates, k_{rel} and $K_{c.m.}$, which are then easily related to the momenta of the two particles, p and q , in order to perform the integration indicated in Eq. (3). Notice that solving the G -matrix equation requires knowledge of the single-particle potential, which in turn requires knowledge of the interaction. Hence, Eqs. (1-2) together with the G -matrix equation constitute a self-consistency problem, which is handled, technically, exactly the same way as previously done for the case of isospin asymmetry [33, 34]. The Pauli operator for scattering of two particles with unequal Fermi momenta, contained in the kernel of the G -matrix equation, is also defined in perfect analogy with the isospin-asymmetric one [34],

$$Q_{\sigma\sigma'}(p, q, k_F^{\sigma}, k_F^{\sigma'}) = \begin{cases} 1 & \text{if } p > k_F^{\sigma} \text{ and } q > k_F^{\sigma'} \\ 0 & \text{otherwise.} \end{cases} \quad (5)$$

The Pauli operator is then expressed in terms of k_{rel} and $K_{c.m.}$ and angle-averaged in the usual way.

III. THE SPLITTING OF THE ONE-BODY POTENTIAL IN SPIN-POLARIZED NUCLEAR MATTER

Figure 1 displays the average potential energy of nucleons, $< U_{u/d} >$, in polarized SNM as a function of the degree of spin asymmetry, described by the spin-asymmetry parameter $\alpha = \frac{\rho_u - \rho_d}{\rho_u + \rho_d}$ ($\alpha > 0$, allowing the spin-up species to

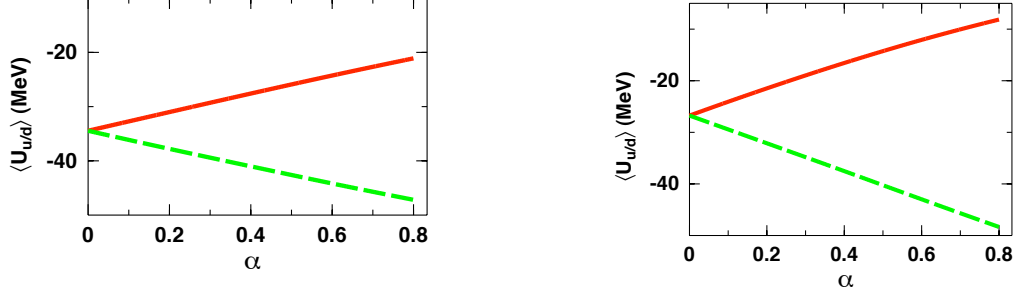


FIG. 1: (color online) The spin splitting of the *average* potential energy in polarized nuclear matter as a function of the spin asymmetry parameter. The average is taken over three-dimensional momenta. The (average) Fermi momentum is equal $1.4 fm^{-1}$ (left frame) and $1.6 fm^{-1}$ (right frame).

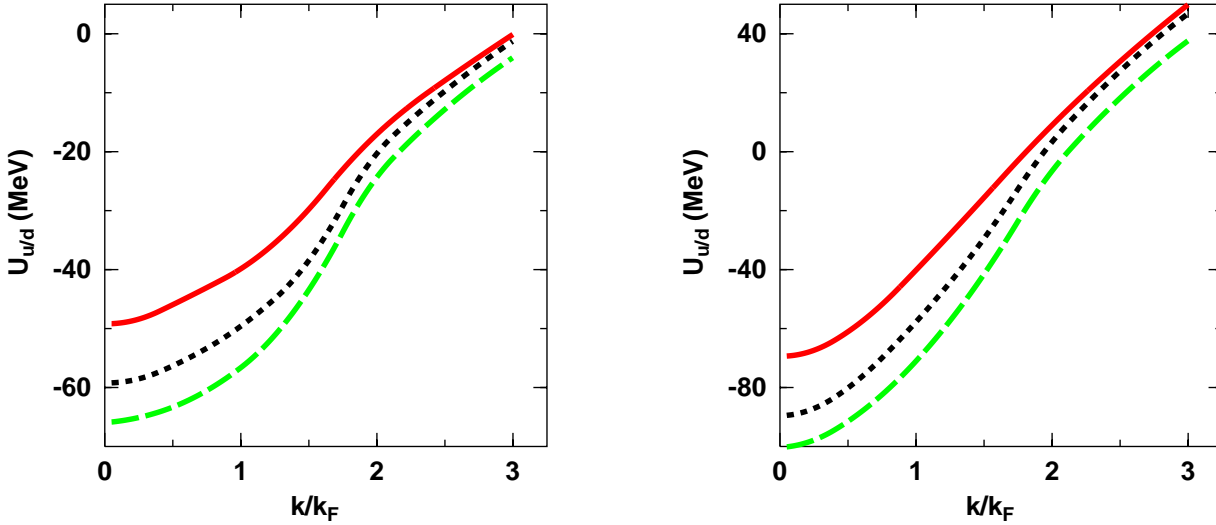


FIG. 2: (color online) The momentum dependence of the single-nucleon potential with spin up (highest curve) and spin down (lowest curve) in spin-asymmetric matter with $\alpha=0.6$. The middle curve displays the potential in spin saturated matter. The Fermi momentum is equal to $1.1 fm^{-1}$ in the left frame and $1.4 fm^{-1}$ in the right frame.

increase in density). The splitting becomes more pronounced with increasing density, compare the left and right panels in the figure. As mentioned earlier, these potentials become direction-dependent in the presence of spin asymmetry, although we found such dependence (on the polar angle θ) to be very mild. In Fig. 1, the potentials are averaged with respect to both magnitude and direction of the momenta.

From the approximate linear relation apparent from Fig. 1, one can write

$$\langle U_{u/d}(\rho, \alpha) \rangle \approx \langle U(\rho) \rangle_0 \pm \langle U_{sym}^S(\rho) \rangle \alpha, \quad (6)$$

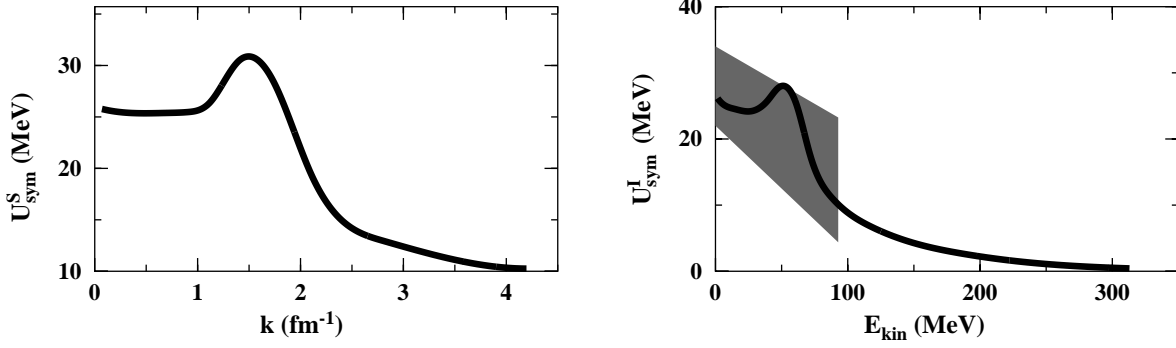


FIG. 3: Left frame: the spin symmetry potential as a function of the momentum. The Fermi momentum is equal to $1.4 fm^{-1}$. The frame on the right shows the symmetry potential in isospin asymmetric (unpolarized) matter at the same density [33]. The shaded area inside the right panel represents empirical constraints from Ref. [30].

where $\langle U_{sym}^S \rangle$ plays the role of an (average) “spin symmetry” potential,

$$\langle U_{sym}^S \rangle = (\langle U_u \rangle - \langle U_d \rangle) / 2\alpha. \quad (7)$$

For a more direct connection with an actual physical experiment, one would write (suppressing, for simplicity, ρ and α dependences)

$$U_{u/d} = U_0 + \mathcal{U}_\sigma \frac{\vec{s} \cdot \vec{\Sigma}}{A}, \quad (8)$$

where \vec{s} and $\vec{\Sigma}$ are the projectile spin and the expectation value of the target spin operator, respectively, and A is the mass number of the target. (The momentum dependence may or may not be taken into account, depending on the particular analysis.) Because

$$\frac{\vec{s} \cdot \vec{\Sigma}}{A} = \frac{1}{A} \frac{1}{2} \sigma_z \left(\frac{1}{2} N_u - \frac{1}{2} N_d \right), \quad (9)$$

and $(N_u - N_d)/A$ is easily identified as the parameter α in the neutron-rich nucleus, one can establish an obvious relation between \mathcal{U}_σ of Eq. (8) and U_{sym}^S defined as in Eq. (6) (without average if the momentum dependence is being analyzed). In practice, a spin unsaturated nucleus will also have a net isospin, which means that \mathcal{U}_σ , \mathcal{U}_τ , and $\mathcal{U}_{\sigma\tau}$ would all have to be considered. Comparison with some older analyses, (based, mostly, on proton scattering on ^{27}Al and ^{59}Co and neutron scattering on ^{59}Co), was performed in Refs. [35, 36].

Next, we display the momentum dependence of $U_u(k)$ and $U_d(k)$ at some fixed values of α and for fixed density. The polar angle is also kept fixed (at the value of $\theta=0$) in view of the mild angular dependence mentioned earlier. Again, we see how the spin-up and the spin-down potentials become more repulsive and more attractive, respectively. It is interesting to analyze the reasons for this behavior, as it sheds light on the similarity between spin and isospin asymmetries. First, let us assume, for the sake of simplification, that k_F^u is much larger than k_F^d , so that U_u and U_d get the largest contributions from the U_{uu} and the U_{du} terms, respectively (which have the same, larger, integration limit, k_F^u). Thus, $U_u - U_d \approx U_{uu} - U_{du}$. The U_{uu} term receives contribution only from the $S=1, M_S=\sigma+\sigma'=+1$ matrix elements. Moving on to the U_{du} term, it receives contributions, with equal weights, from $S=0, M_S=0$ and from $S=1, M_S=0$ matrix elements. When all of the appropriate weighting factors are taken into account, the interaction

among nucleons with like spin projections turns out to be *more repulsive* than the one among nucleons with opposite spin components. Thus, the scenario becomes analogous to the case of isospin-asymmetric nuclear matter, where the interaction among like nucleons (with total isospin equal to 1), is more repulsive than the one among neutrons and protons. (It may be useful to mention that all arguments would remain invariant upon exchange of “u” and “d” labels. The physical source of the splitting we observe is in the different nature of the nuclear force between nucleons with parallel or antiparallel spins.)

The “spin symmetry potential”, $U_{sym}^S = (U_u - U_d)/(2\alpha)$, is displayed in Fig. 3 as a function of the momentum. It is remarkably similar, both qualitatively and quantitatively, to the symmetry potential for IANM [33] shown on the RHS of the figure, $U_{sym}^I = (U_n - U_p)/(2\alpha)$. Concerning the latter, it has been shown that, starting from a phenomenological formalism for the single-nucleon potential, it is possible to predict opposite tendencies for the energy dependence of the symmetry potential (while still maintaining nearly the same value of the symmetry energy [37–39]), resulting in very different predictions of some heavy-ion observables. Similar uncertainties are to be expected with U_{sym}^S .

Finally, we notice that the approximately linear dependence (*vs.* α) manifest from Fig. 1, along with a similar behavior of the kinetic energy, implies the well-known parabolic form for the EoS:

$$< e(\rho, \alpha) > \approx < e_0(\rho) > + < e_{sym}^S(\rho) > \alpha^2. \quad (10)$$

This is demonstrated in Fig. 4, where the energy/particle (averaged over spin-up and spin-down nucleons) is compared with the parabolic approximation.

Before closing, we stress again the importance of more and better empirical constraints to gain insight into the spin-dependent part of the nucleon-nucleus optical potential, and, thus, the spin-dependent nuclear effective interaction. Valuable information can also come from heavy-ion collisions, provided polarized heavy targets are available.

Within the Landau theory of a Fermi liquid, the effective quasiparticle interaction is represented in terms of functions associated with the various spin and isospin operators. The Landau parameters are the lowest order terms in the Legendre polynomial expansions of those functions. In particular, the strength of the interaction associated with the $\sigma_1 \cdot \sigma_2$ operator is represented, to lowest order, by the g_0 Migdal-Landau parameter, which drives nuclear matter instabilities against spin fluctuations. Thus, stringent constraints on the latter, including its density dependence, would provide much-needed insight into spin-spin correlations and the possibility of such instabilities. Because the expectation value of the $\sigma_1 \cdot \sigma_2$ operator is equal to -3 in the singlet states and +1 in the triplet states, values of g_0 which decrease with increasing density would signify that the spin-spin force turns less attractive in the singlet configuration and less repulsive in the triplet one. Thus, there may come a point (in terms of density) when the state with aligned spins is energetically more favorable than the unpolarized one, resulting in spin instability.

IV. CONCLUSIONS

We continue our broad analysis of various phases of nuclear matter. Here, we specifically address the splitting of the single-nucleon potential in spin-polarized, but isospin-symmetric, nuclear matter. The behavior of the predictions is perfectly parallel to the one encountered in IANM. We point out that additional constraints are crucial for a better understanding of the polarizability of nuclear matter. Spin and isospin unsaturated phases of nuclear matter are also interesting systems which we plan to study, although computationally more involved.

As usual, we adopt the microscopic approach for our nuclear matter calculations. Concerning our many-body method, we find DBHF to be a good starting point to look beyond the normal states of nuclear matter, which it describes successfully. The main strength of this method is its inherent ability to effectively incorporate crucial TBF contributions through relativistic effects (see Ref. [33] and references therein).

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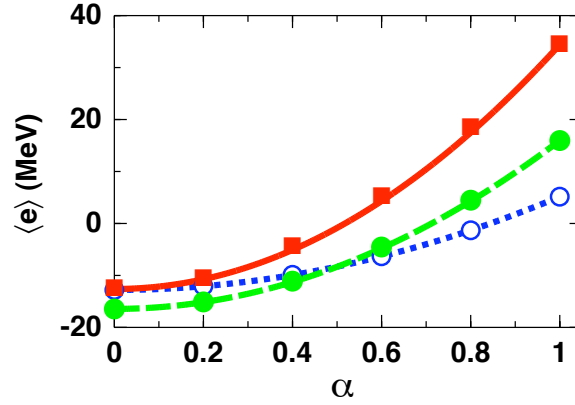


FIG. 4: (color online) The energy/particle in polarized SNM at three fixed densities as a function of the spin asymmetry parameter. The solid lines are the parabolic approximation, Eq. (10), to the calculated values. The various densities are in units of fm^{-3} .

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